22/4/2020 Wed. (9)

Base :- Let $(X, \mathfrak{I})$ be a topological space.
Let $\mathcal{B} \subset \mathfrak{I}$ s.t. $\mathcal{B} \neq \phi$.
$\mathcal{B}$ is said to be a base or open base or basis for the topology $\mathfrak{I}$ on X if for any non-empty set $G \in \mathfrak{I} \Rightarrow \exists\ B_1 \subset \mathcal{B}$ s.t. $\boxed{G = \cup\{B : B \in \mathcal{B}\}}$

Ex :- The set of all open intervals in R form a base for the usual topology on R

(ii) **Local Base** :- Let $(X, \mathfrak{I})$ be a topological space. A family B_x of open subsets of X is said to be a local base at $x \in X$ for the topology $\mathfrak{I}$ on X if

(i) $B \in B_x \Rightarrow x \in B$
(ii) any $G \in \mathfrak{I}$ with $y \in G \Rightarrow B \in B_x$ st $y \in B \subset G$

(iii) **First Countable Space** :- Let $(X, \mathfrak{I})$ be a topological space. The space X is said to satisfy the first axiom of countability if X has a countable base at each $x \in X$. Then X is called first countable space

(iv) **Second countable space** :- Let $(X, \mathfrak{I})$ be a topological space. The space X is said to satisfy second axiom of countability if $\exists$ a countable base for $\mathfrak{I}$ on X.

Ex :- The set of all open intervals (r, s) with r and s as rational numbers form a base.
(R, U) is second countable.

Thm :- A second countable space is always first countable space.
Thm :- first countable space does not imply second countable space.

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Hereditary Property :- Let $(X, \mathfrak{J})$ be a topological space. A property of X is said to be hereditary if the property is possessed by every subspace of X, e.g. first countable space, second countable space but closed sets, open sets are not hereditary property.

Some more about Base for a Topology

Let $(X, \mathfrak{J})$ be a topological space on X and $\mathcal{B} \subset \mathfrak{J}$, then we have

(i) each $G \in \mathfrak{J}$ is the union of member of $\mathcal{B}$

(ii) for any $x \in G$ $\exists$ $B \in \mathcal{B}$ with

$x \in B \subset G$