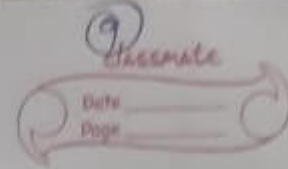


22/4/2020 Wed



Base :- Let (X, τ) be a topological space.

Let $\mathcal{B} \subset \tau$ st. $\mathcal{B} \neq \emptyset$.

\mathcal{B} is said to be a base or open base or basis for the topology τ on X if for any non-empty set $G \in \tau \Rightarrow \exists \mathcal{B}_x \subset \mathcal{B}$ st. $G = \bigcup \{B : B \in \mathcal{B}_x\}$

Ex :- The set of all open intervals in \mathbb{R} form a base for the usual topology on \mathbb{R} .

(i) Local Base :- Let (X, τ) be a topological space.

A family \mathcal{B}_x of open subsets of X is said to be a local base at $x \in X$ for the topology τ on X if

(i) $B \in \mathcal{B}_x \Rightarrow x \in B$

(ii) any $G \in \tau$ with $x \in G \Rightarrow \exists B \in \mathcal{B}_x$ st. $x \in B \subset G$

(ii) First Countable Space :- Let (X, τ) be a topological space. The space X is said to satisfy the first axiom of countability if X has a countable base at each $x \in X$. Then X is called first countable space.

(iii) Second countable space :- Let (X, τ) be a topological space. The space X is said to satisfy second axiom of countability if \exists a countable base for τ on X .

Ex :- The set of all open intervals (r, s) with r and s as rational numbers form a base. (\mathbb{R}, τ) is second countable.

Theorem :- A second countable space is always first countable space.

Theorem :- first countable space does not imply second countable space.

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Hereditary Property :- Let (X, τ) be a topological

space. A property of X is said to be hereditary if the property is possessed by every subspace of X , e.g. first countable space, second countable space but closed sets, open sets are not hereditary property.

Some more about Base for a Topology

Let (X, τ) be a topological space on X and $\mathcal{B} \subset \tau$, then we have

(i) each $G \in \tau$ is the union of members of \mathcal{B}

(ii) for any $x \in G$ $\exists B \in \mathcal{B}$ with

$$\underline{x \in B \subset G}$$